

# GAUGE SYMMETRY OF THE THIRD KIND AND QUANTUM MECHANICS AS AN INFRARED PHENOMENON

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We introduce functional degrees of freedom by a new gauge principle related to the phase of the wave functional. Thereby, quantum mechanical systems are seen as dissipatively embedded part of a nonlinear classical structure producing universal correlations. There are a fundamental length and an entropy/area parameter, besides standard couplings. For states that are sufficiently spread over configuration space, quantum field theory is recovered.

*It is a great pleasure to participate in the celebration and an honour to contribute this article to the Festschrift for Adriano Di Giacomo on occasion of his 70th birthday: With my very best wishes!*

## 1 Introduction

We consider  $\mathcal{U}(1)$  gauge transformations “of the third kind” which are restricted only to be local in the space of field configurations underlying quantum field theory (QFT).

Our point of departure is the common observation that the potentiality described by the wave function(al)  $\Psi$  and the reduction to the actuality of the outcome of any measurement process have been left outside of standard quantum theory:

“... , it is an incomplete representation of real things, although it is the only one which can be built out of the fundamental concepts of force and material points (quantum corrections to classical mechanics). The incompleteness of the representation leads necessarily to the statistical nature (incompleteness) of the laws.”<sup>1</sup>

It appears that aspects of the theory that concern the concept of information have hitherto been left separate from the concepts of force and material points, the “real things” Einstein refers to. This is reflected, on the other side, in the remarkable derivation of the kinematical setting of quantum theory from three information theoretical constraints, as discussed, for example, in the review by Bub<sup>2</sup>: the “real things” do not play a role in it.

One cannot help to feel that the cut between “real things” and “information about real things” is due to historical contingencies, not unlike the cut between dynamical theories, describing the effects of gravity in particular, and geometrical theories of space and time, until their fusion in general relativity.

Reconsidering the role of the wave functional, our gauge principle is based on the assumption that its phase is not only subject to global gauge transformations (“of the first kind”) and local ones (“of the second kind”) related to its variables, which are the common fields of QFT. We introduce another class of functional gauge transformations which are *local in the space of field configurations*, attributing a physical ‘charge’ to  $\Psi$ .

Guided by the gauge principle, we couple a dynamical functional  $\mathcal{A}$  to the wave functional  $\Psi$ . We generalize the action of Dirac’s variational principle, as applied to QFT, which is useful to study the interplay with the usual symmetries. Here, the would-be-quantum sector described by  $\Psi$  forms a dissipative subsystem of the enlarged structure, effectively extending it nonlinearly and nonlocally in the space of field configurations.

We recall that much work has been done on nonlinear extensions of nonrelativistic quantum mechanics, most notably by Bialynicki-Birula and Mycielski<sup>3</sup> and by Weinberg<sup>4</sup>. – Works of Kibble<sup>5</sup> and of Kibble and Randjbar-Daemi<sup>6</sup> present nonlinear extensions of scalar QFT models, the latter coupled to gravity, where parameters of the models are quantum state dependent. While this kind of nonlinearity is not compatible with our gauge principle, common feature is, of course, the effective *nonlinearity* (in  $\Psi$ ) of the wave functional equation.

The local U(1) gauge invariance of the Schrödinger equation in quantum mechanics (“first quantization”) leads to the electromagnetic interaction of charged particles via the classical minimal coupling prescription. Here, we explore an analogous dynamical scheme for the functional Schrödinger equation (“second quantization”). It predicts the universal coupling of *all* fields that are variables in this equation and necessitates the introduction of a fundamental length.

In this way, we obtain an embedding of quantum theory into an apparently classical framework, where the potentiality represented by  $\Psi$  bears the character of a charge. It causes correlations of the underlying fields beyond what is encoded in their usual Lagrangians, with QFT describing an infrared limit. It is tempting to speculate that a model of this kind might point in the right direction towards putting the concepts of “real things” and “information about real things” on a common footing.

In distinction to recent attempts to reconstruct quantum mechanics as an emergent theory<sup>7,8,9,10,11,12,13,14</sup>, the present approach, so far, does not depend on a particular field theory, such as the Standard Model. We share, however, the tentative conclusion that quantum theory can and should be reconstructed as an effective theory describing large-scale behavior of *fundamentally deterministic degrees of freedom*<sup>15</sup>. Quantum states are no longer the primary degrees of freedom. Bell’s theorem and the predicament of local hidden variable theories are circumvented, since the implicit nonlocality operates only at the pre-quantum level.

In Section 2, we recall the minimally coupled Schrödinger equation, in order to contrast it with the functional case in what follows. We stress that it amounts to introducing the nonrelativistic limit of a *classical* charged scalar field theory incorporating the electromagnetic interaction. This is relevant for the interpretation of our approach, where the wave functional  $\Psi$  itself carries the new universal U(1) ‘charge’ that acts as a source of the corresponding functional ‘field’  $\mathcal{A}$ . When the latter becomes negligible, ordinary QFT is recovered. Formal aspects are developed in Section 3 for the case of a single scalar field, while the extension incorporating other matter and gauge fields is briefly indicated and will be reported in detail elsewhere. We conclude with a list of problems and speculations left for further study.

## 2 Minimal Coupling in the Schrödinger Equation

Textbooks expand on the topic of this section; for example, see Ref.<sup>16</sup>, which contains clear discussions of the foundations of quantum mechanics and of its interpretation. We represent it here, in order to distinguish the new element of the following section.

We consider the single-particle Schrödinger equation in the coordinate representation:

$$i\partial_t\psi(\vec{x},t) = H(\vec{p} \rightarrow \frac{1}{i}\nabla, \vec{x})\psi(\vec{x},t) = \left\{ -\frac{1}{2m}\nabla^2 + V(\vec{x}) \right\}\psi(\vec{x},t) \quad , \quad (1)$$

with the canonical momentum in the Hamiltonian  $H$  replaced by the derivative operator,  $V$  representing an external potential, and  $m$  denoting the particle mass, as usual. Units are chosen such that  $\hbar = c = 1$ .

The equation is invariant under *global* gauge transformations (“of the first kind”) of the form  $\psi' = \exp(-i\lambda)\psi$ ,  $\lambda$  real. Correspondingly, there is the continuity equation:

$$\partial_t(\psi^*\psi) + \nabla \cdot (\psi^*\nabla\psi - \psi\nabla\psi^*)/2mi = 0 \quad , \quad (2)$$

which expresses the local *probability conservation*, according to the probability amplitude interpretation of the wave function.

Next, we consider *local* gauge transformations (“of the second kind”):

$$\psi'(\vec{x}, t) = \exp(-ie\lambda(\vec{x}, t)) \psi(\vec{x}, t) \quad , \quad (3)$$

with the coupling constant  $e$  and  $\lambda$  a real function. The Schrödinger equation (1) remains invariant, provided all derivatives are replaced by covariant derivatives,

$$\partial_\mu \equiv (\partial_t, \nabla) \rightarrow D_\mu \equiv \partial_\mu + ieA_\mu \equiv (\partial_t + ieA^0, \nabla - ie\vec{A}) \quad , \quad (4)$$

and provided the vector potential transforms as

$$A'_\mu(x) = A_\mu(x) + \partial_\mu\lambda(x) \quad , \quad (5)$$

with  $x^\mu \equiv (t, \vec{x})$ , considering Minkowski space with metric  $g_{\mu\nu} \equiv \text{diag}(1, -1, -1, -1)$ . The vector potential describes the electromagnetic field,  $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$ , which is classical here. The related *current conservation* law is obtained from Eq. (2) using the substitution (4) and an overall multiplication by  $e$ .

While all of this is familiar, two important remarks are in order:

(A) The quantum mechanical model of a charged particle interacting with the electromagnetic field descends from the classical Maxwell theory, via the minimally coupled classical particle Hamiltonian, through its quantization, finally to the gauge invariant Schrödinger equation. The point to make is that there is a *classical* regime where the quantum theory is anchored, in agreement with the Copenhagen interpretation<sup>16</sup>.

(B) The free Schrödinger equation presents the nonrelativistic limit of the Klein-Gordon equation. The latter is not acceptable as a quantum mechanical single-particle wave equation, since there would be negative energy states and a ‘probability’ density which is not positive definite<sup>16</sup>. However, with minimal coupling, the Klein-Gordon equation describes *interacting classical (!) scalar and electromagnetic fields*. (Of course, quantization then leads to “second quantized” scalar electrodynamics.)

In the following section, we show that QFT in the functional Schrödinger picture, even with all known fields and interactions included, can be extended in analogy to the Klein-Gordon equation by incorporating a functional  $\mathcal{U}(1)$  gauge symmetry (“of the third kind”). This may bring it close to a deterministic pre-quantum theory.

### 3 The Gauge Invariant Functional Wave Equation

The functional Schrödinger picture of QFT is best suited for our argument. It is intuitively appealing by the resemblance to the case of first quantized systems. We refer to Refs.<sup>17,18,19,20</sup> for reviews, several applications, and further references.

### 3.1 The Scalar Field Case

Beginning with a generic scalar field theory, the functional Schrödinger equation is:

$$i\partial_t \Psi[\varphi; t] = H[\hat{\pi}, \varphi] \Psi[\varphi; t] \equiv \int d^3x \left\{ -\frac{1}{2} \frac{\delta^2}{\delta \varphi^2} + \frac{1}{2} (\nabla \varphi)^2 + V(\varphi) \right\} \Psi[\varphi; t] , \quad (6)$$

where the Hamiltonian corresponds to the classical Hamiltonian density, including mass and selfinteraction terms in  $V(\varphi)$ . Implementing the quantization, the classical canonical momentum conjugate to the field (coordinate)  $\varphi$  has been substituted according to:

$$\pi(\vec{x}) \longrightarrow \hat{\pi}(\vec{x}) \equiv \frac{1}{i} \frac{\delta}{\delta \varphi(\vec{x})} , \quad (7)$$

i.e., in this coordinate representation,  $[\varphi(\vec{x}), \hat{\pi}(\vec{y})] = i\delta^3(\vec{x} - \vec{y})$ , as it should be.

In close analogy to gauge transformations in the first quantized Schrödinger picture, we now introduce the  $\mathcal{U}(1)$  *gauge transformation of the third kind*:

$$\Psi'[\varphi; t] = \exp(-if\Lambda[\varphi; t]) \Psi[\varphi; t] , \quad (8)$$

where  $\Lambda$  denotes a time dependent real functional and  $f$  a new dimensionless coupling constant. Note that this gauge transformation is local in the space of field configurations. The difference from the usual gauge transformations in QFT will show up in the way we introduce covariant derivatives (see also Section 3.3).

In fact, the wave functional equation (6) can be made invariant under the transformation (8) by replacing derivatives by covariant ones:

$$\partial_t \longrightarrow \mathcal{D}_t \equiv \partial_t + if\mathcal{A}_t[\varphi; t] , \quad (9)$$

$$\frac{\delta}{\delta \varphi(\vec{x})} \longrightarrow \mathcal{D}_{\varphi(\vec{x})} \equiv \frac{\delta}{\delta \varphi(\vec{x})} + if\mathcal{A}_{\varphi}[\varphi; t, \vec{x}] . \quad (10)$$

The real functional  $\mathcal{A}$  presents the ‘potential’ or ‘connection’, analogous to the vector potential in Eqs. (4). Generally,  $\mathcal{A}$  depends on  $t$ . However, it is a *functional* of  $\varphi$  in Eq. (9), while it is a *functional field* in Eq. (10); the latter includes fields as a special case, for example,  $\mathcal{A}_{\varphi}[\varphi; t, \vec{x}] = \mathcal{A}(\varphi(\vec{x}); t)$ . We distinguish these different components of  $\mathcal{A}$  by the subscripts. Furthermore, the ‘potentials’ need to transform according to:

$$\mathcal{A}'_t[\varphi; t] = \mathcal{A}_t[\varphi; t] + \partial_t \Lambda[\varphi; t] , \quad (11)$$

$$\mathcal{A}'_{\varphi}[\varphi; t, \vec{x}] = \mathcal{A}_{\varphi}[\varphi; t, \vec{x}] + \frac{\delta}{\delta \varphi(\vec{x})} \Lambda[\varphi; t] , \quad (12)$$

under the gauge transformation. Then, we may also define a ‘field strength’:

$$\mathcal{F}_{t\varphi}[\varphi; t, \vec{x}] \equiv \partial_t \mathcal{A}_{\varphi}[\varphi; t, \vec{x}] - \frac{\delta}{\delta \varphi(\vec{x})} \mathcal{A}_t[\varphi; t] , \quad (13)$$

which is invariant under the transformations (11), (12); note that  $\mathcal{F}_{t\varphi} = [\mathcal{D}_t, \mathcal{D}_{\varphi}]/if$ .

In order to give a meaning to the coupling constant  $f$ , we have to postulate a consistent dynamics for the gauge potential  $\mathcal{A}$ . All elementary fields supposedly are present as the

coordinates on which the wave functional depends – just a scalar field, besides time, in this section. Then, we may consider the following  $\mathcal{U}(1)$  invariant action:

$$\Gamma \equiv \int dt D\varphi \left\{ \Psi^* \left( \mathcal{N}(\rho) \overleftrightarrow{i\mathcal{D}_t} - H\left[\frac{1}{i}\mathcal{D}_\varphi, \varphi\right] \right) \Psi - \frac{l^2}{2} \int d^3x (\mathcal{F}_{t\varphi})^2 \right\} , \quad (14)$$

where  $\Psi^* \mathcal{N} \overleftrightarrow{i\mathcal{D}_t} \Psi \equiv \frac{1}{2} \mathcal{N} \{ \Psi^* i\mathcal{D}_t \Psi + (i\mathcal{D}_t \Psi)^* \Psi \}$ , and with a dimensionless real function  $\mathcal{N}$  depending on the density:

$$\rho[\varphi; t] \equiv \Psi^*[\varphi; t] \Psi[\varphi; t] . \quad (15)$$

We shall see shortly that  $\mathcal{N}$  incorporates a necessary *nonlinearity*. The fundamental parameter  $l$  has dimension  $[l] = [length]$ , for dimensionless measure  $D\varphi$  and  $\Psi$ .

Our action generalizes the action for the wave functional of a scalar field, which has been employed for applications of Dirac's variational principle to QFT, e.g., in Refs.<sup>6,21</sup>. The quadratic part in  $\mathcal{F}_{t\varphi}$  is the simplest possible extension, i.e. local in  $\varphi$  and quadratic in the derivatives, besides the newly introduced nonlinearity in  $\rho$ . – The action depends on  $\Psi$ ,  $\Psi^*$ ,  $\mathcal{A}_t$ , and  $\mathcal{A}_\varphi$  separately. While a Hamiltonian formulation is possible, we will obtain the equations of motion and a constraint directly by varying  $\Gamma$  with respect to the variables. We assume that all necessary (functional) partial integrations are justified.

Thus, varying  $\Gamma$  with respect to  $\Psi^*$  (and  $\Psi$ ) yields the gauge invariant  $\Psi$ -functional equation of motion (and its adjoint):

$$(\rho \mathcal{N}(\rho))' i\mathcal{D}_t \Psi[\varphi; t] = H\left[\frac{1}{i}\mathcal{D}_\varphi, \varphi\right] \Psi[\varphi; t] , \quad (16)$$

replacing the Schrödinger equation (6); here  $f'(\rho) \equiv df(\rho)/d\rho$ . Varying with respect to  $\mathcal{A}_\varphi$ , we obtain the invariant ‘gauge field equation’:

$$\partial_t \mathcal{F}_{t\varphi}[\varphi; t, \vec{x}] = \frac{f}{2il^2} \left( \Psi^*[\varphi; t] \mathcal{D}_{\varphi(\vec{x})} \Psi[\varphi; t] - \Psi[\varphi; t] (\mathcal{D}_{\varphi(\vec{x})} \Psi[\varphi; t])^* \right) , \quad (17)$$

which completes the set of dynamical equations.

The nonlinear Eq. (16) preserves the normalization, i.e. any imposed value of  $\langle \Psi | \Psi \rangle \equiv \int D\varphi \Psi^* \Psi$  is conserved, while the overlap of two different states,  $\langle \Psi_1 | \Psi_2 \rangle$ , generally varies in time. While this may hint at a probability interpretation, the continuity equation, Eq. (19) below, shows that this cannot be maintained, in general. For  $\mathcal{A} \neq 0$ , also the *homogeneity property* does no longer hold, i.e.,  $\Psi$  and  $z\Psi$  ( $z \in \mathbf{Z}$ ) present two different physical states<sup>3,4</sup>. This changes an essential aspect of the measurement theory<sup>5</sup> and indicates that here QFT is embedded in a *classical* framework (see Section 3.2).

Furthermore, note that the Hamiltonian  $H$ , unlike in QFT, cannot be arbitrarily shifted by a constant  $\Delta E$ , transforming  $\Psi \rightarrow \exp(-i\Delta E t) \Psi$ . Our action is, however, invariant under space-time translations and spatial rotations. The behavior under Lorentz boosts is more difficult to assess (cf. Ref.<sup>5</sup>) and will be considered elsewhere.

In the action there is no time derivative acting on the variable  $\mathcal{A}_t$  which, therefore, acts as a Lagrange multiplier for a constraint. Thus, variation with respect to  $\mathcal{A}_t$  yields the corresponding gauge invariant ‘Gauss’ law’:

$$- \int d^3x \frac{\delta}{\delta \varphi(\vec{x})} \mathcal{F}_{t\varphi}[\varphi; t, \vec{x}] = \frac{f}{l^2} \Psi^*[\varphi; t] \Psi[\varphi; t] \mathcal{N}(\rho) . \quad (18)$$

This can be combined with Eq. (17) to result in the continuity equation:

$$0 = \partial_t (\rho \mathcal{N}(\rho)) + \frac{1}{2i} \int d^3x \frac{\delta}{\delta \varphi(\vec{x})} \left( \Psi^* \mathcal{D}_{\varphi(\vec{x})} \Psi - \Psi (\mathcal{D}_{\varphi(\vec{x})} \Psi)^* \right) , \quad (19)$$

which expresses the local  $\mathcal{U}(1)$  ‘charge’ conservation in the space of field configurations.

Furthermore, functionally integrating Eq. (18), we find that the total ‘charge’  $Q$  has to vanish at all times:

$$Q(t) \equiv \frac{f}{l^2} \int D\varphi \rho \mathcal{N}(\rho) = 0 , \quad (20)$$

since the functional integral of a total derivative is zero. Here, the necessity of the nonlinearity becomes obvious. Without it, the vanishing total ‘charge’ could not be implemented. However, besides necessarily multiplying the invariant term  $\Psi^* i \mathcal{D}_t \Psi$  in the action, the nonlinearity is still undetermined.

Mean field type nonlinearities have been considered in Refs.<sup>5,6</sup> before, with nonlinear modifications of the underlying Hamiltonian  $H$ . They are based on a field  $\bar{\rho}$ :

$$\bar{\rho}(\vec{x}) \equiv \frac{\int D\varphi \Psi^*[\varphi; t] \varphi^n(\vec{x}) \Psi[\varphi; t]}{\int D\varphi' \Psi^*[\varphi'; t] \Psi[\varphi'; t]} , \quad (21)$$

e.g., with  $n = 2$ , which here is  $\mathcal{U}(1)$  invariant. Nevertheless, this type of nonlinearity leads to a conflict with the condition of vanishing total charge.

Anticipating some justification by the following analysis, we presently choose:

$$\rho \mathcal{N}(\rho) \equiv \rho \log(\rho) + \rho S , \quad (22)$$

with  $S$  a dimensionless parameter. We consider  $\rho \log(\rho)$  as the negative of an *entropy density*, recalling that the density  $\rho$  can be normalized,  $\int D\varphi \rho = 1$ , independent of time. Furthermore, it should be replaced by a proper *probability* in this entropy term:

$$\rho_P[\varphi] \equiv \int_{\varphi - \Delta\varphi/2}^{\varphi + \Delta\varphi/2} D\varphi' \rho[\varphi'] , \quad (23)$$

i.e., the probability to find the system within an interval  $\Delta\varphi$  around the field configuration  $\varphi$ . The natural scale for  $\Delta\varphi$  is  $l^{-1}$ . If  $\rho$  is essentially constant over such an interval, then the coarse-graining leading to  $\rho_P$  simply amounts to multiplication by its volume, which needs to be regularized. We keep this in mind and work with Eq. (22).

With this choice, the time derivative of  $\Psi$  in Eq. (16) is multiplied by:

$$(\rho \mathcal{N}(\rho))' = 1 + S + \log(\rho) . \quad (24)$$

Thus, the timescale of the  $\Psi$ -functional evolution effectively shrinks or expands, depending on the probability to find the system in the respective region of configuration space.

We conclude this section with several observations:

(A) The dimensionless coupling constant  $f$  can be absorbed by rescaling:

$$\mathcal{A} \longrightarrow f^{-1} \mathcal{A} . \quad (25)$$

The resulting factor  $f^{-2}$  multiplying  $l^2$  in the action simply redefines this parameter. Consequently, we set  $f = 1$  from now on. – Furthermore, our equations involve second

functional derivatives at coinciding points which should be regularized. We do not perform this here and proceed heuristically.

(B) The system of Eqs. (16)–(18), and (22) obeys a *weak superposition principle*<sup>3</sup>: The sum of two solutions,  $\Psi_{1,2}$ , that do not overlap, presents also a solution, provided that  $\mathcal{A} = \mathcal{A}_1 + \mathcal{A}_2$  is determined consistently.

(C) It seems questionable whether our nonlinear extension of QFT is *local* in the usual sense<sup>5</sup>: Leaving only  $i\partial_t\Psi$  on the left-hand side of Eq. (16), the resulting evolution operator on the other side cannot be written as integral of a density  $\tilde{H}(\vec{x})$ , in order to check whether  $[\tilde{H}(\vec{x}), \tilde{H}(\vec{x}')] = 0$ , for  $\vec{x} \neq \vec{x}'$ , meaning locality. Yet, instead, the system may be perturbed by a small source added to the action, such as  $\int dt D\varphi \Psi^* J[\varphi; t] \Psi$ . This does not modify the ‘gauge field equation’ and ‘Gauss’ law’. If the source is confined to a localized region  $\triangle$  and acts only at  $t = t_0$ ,  $J[\varphi; t] \equiv \delta(t - t_0) \int_{\triangle} d^3x j(\vec{x})\varphi(\vec{x})$ , for example, then  $\Psi[\varphi; t_0 + dt]$  shows no effect when probed by an operator with support  $\{\vec{x}|\vec{x} \notin \triangle\}$ . As usual, only the derivative term  $\mathcal{D}_\varphi^2$  in  $H$  spreads the perturbation (*microcausality*). – However, suppose we integrated out the ‘gauge field’. The resulting effective equation for  $\Psi$  would be *nonlocal in field space and in space-time*.

(D) We find from Eqs. (18), (22) the interesting result that the ‘charge’ density  $\rho\mathcal{N}(\rho)$  is the deviation of entropy density per unit area from the reference density  $\rho S/l^2$ . The total entropy is constrained to equal  $S$  by Eq. (20). Therefore, also the *entropy/area*  $S/l^2$  is a parameter here, besides the fundamental *length*  $l$ . – We remark that entropy per area is an essential parameter in seemingly unrelated work of Padmanabhan<sup>22</sup>, where it is suggested that gravity is intrinsically holographic and quantum mechanical.

### 3.2 Stationary States, Separability and QFT Limit

We study the separation of the time dependence in the coupled Eqs. (16)–(18), assuming:

$$\Psi[\varphi; t] \equiv e^{-i\omega t} \Psi_\omega[\varphi] \quad , \quad (26)$$

with  $\omega \in \mathbf{R}$ , and consistently assume *time independent*  $\mathcal{A}$ -functionals. With the nonlinearity of Eq. (22), we obtain from Eq. (16):

$$\left(1 + S + \log(\rho_\omega)\right) \left(\omega - \mathcal{A}_t[\varphi]\right) \Psi_\omega[\varphi] = H\left[\frac{1}{i}\mathcal{D}_\varphi, \varphi\right] \Psi_\omega[\varphi] \quad , \quad (27)$$

with  $\mathcal{D}_\varphi = \frac{\delta}{\delta\varphi} + i\mathcal{A}_\varphi$  and  $\rho_\omega \equiv \Psi_\omega^*[\varphi] \Psi_\omega[\varphi]$ . From Eq. (17) follows:

$$\frac{1}{2i} \left( \Psi_\omega^*[\varphi] \mathcal{D}_{\varphi(\vec{x})} \Psi_\omega[\varphi] - \Psi_\omega[\varphi] (\mathcal{D}_{\varphi(\vec{x})} \Psi_\omega[\varphi])^* \right) = 0 \quad , \quad (28)$$

which expresses the vanishing of the ‘current’ in the stationary situation.

Applying a time independent gauge transformation, cf. Eqs. (8), (12), the stationary wave functional can be made *real*. The Eq. (28) then implies  $\mathcal{A}_\varphi = 0$ ; consequently, we replace  $\mathcal{D}_\varphi \rightarrow \frac{\delta}{\delta\varphi}$  everywhere. Finally, ‘Gauss’ law’, Eq. (18), determines  $\mathcal{A}_t$ :

$$\int d^3x \frac{\delta^2}{\delta\varphi(\vec{x})^2} \mathcal{A}_t[\varphi] = \frac{1}{l^2} \rho_\omega \left( S + \log(\rho_\omega) \right) \quad , \quad (29)$$

incorporating Eq. (22). This has to be solved selfconsistently together with Eq. (27).

Separation of the time dependence thus leads to two coupled equations. One may guess an appropriate time independent  $\Psi_\omega$ -functional, for example some kind of Gaussian. Having an action at hand, Eq. (14), the parameters of such an Ansatz can be optimized according to the variational principle. This is analogous to Hartree approximation and semiclassical limit of QFT<sup>19,21</sup>. Furthermore, the Eq. (29) can be solved, at least formally, by functional Fourier transformation. In this way, the ‘connection’  $\mathcal{A}_t$  can be eliminated, at the expense of introducing the nonlocality mentioned before.

Let us turn to the question of *separability*. This is an important property of linear quantum theory. It allows to combine subsystems which do not interact with each other, without creating unphysical correlations between them<sup>3,4</sup>. This should be preserved for the phenomena and to the level of accuracy which are accessible experimentally.

We will give a heuristic argument that linear *QFT arises in the infrared (IR) limit*, and consequently separability. Discretizing the system on a finite spatial lattice, would be a first step towards a more rigorous derivation.

Considering the stationary equations, let us assume that the system is in a *diffuse state*, characterized by a density  $\rho_\omega$  that is widely spread over the space of fields  $\varphi$ . In such a high entropy state, we have that  $\rho_\omega$  or, rather, the corresponding probability  $\rho_P[\varphi]$  of Eq. (23) is nearly homogeneous and small everywhere. Introducing the density of the Hamiltonian of Eq. (27), via  $H \equiv \int d^3x H(\vec{x})$ , the local energy density associated with such a (real) state  $\Psi_\omega$ , i.e.  $\epsilon(\vec{x}) \equiv \int D\varphi \Psi_\omega H(\vec{x}) \Psi_\omega$ , must be small, in particular, since also the kinetic term  $\propto \delta_\varphi^2 \Psi_\omega$  cannot be too large (IR limit).

In this situation, assuming  $\rho_\omega \approx \exp(-S)$ , the ‘charge density’ on the right-hand side of Eq. (29) is very small. Therefore, we neglect  $\mathcal{A}_t$  compared to  $\omega$  in Eq. (27). Furthermore, the nonlinear prefactor there goes to one in the same limit. Thus, the stationary functional Schrödinger equation results:

$$\omega \Psi_\omega = H \Psi_\omega \quad , \quad (30)$$

and with it the known structures of QFT. – It will be most interesting to explore consequences of the small violations of the linear equation (30), due to terms which involve  $\rho_\omega$  or  $\mathcal{A}_t$ . Similarly as in the mean field type model of Kibble<sup>5</sup>, nonlinear effects become important in our framework only for states with a small uncertainty in configuration space, such that the IR limit does not apply.

Finally, we remark that two stationary solutions,  $\Psi_{\omega_{1,2}}$ , of the present eigenvalue problem, in general, obey an *orthogonality* relation:

$$0 = \int D\varphi \Psi_1^*[\varphi] \Psi_2[\varphi] \left( (1 + S + \log(\rho_{\omega,1}))(\omega_1 - \mathcal{A}_{t1}) - (1 + S + \log(\rho_{\omega,2}))(\omega_2 - \mathcal{A}_{t2}) \right) \quad , \quad (31)$$

where  $\mathcal{A}_{t1,2}$  are determined selfconsistently by Eq. (29). This reduces to the usual one of QFT in the IR limit just discussed.

### 3.3 Internal Symmetries

Some modifications are necessary when the scalar field model of Section 3.1 is replaced by a more realistic field theory of interacting matter and gauge fields. For simplicity, we



restrict ourselves to the Abelian Higgs model or scalar QED<sup>19</sup>. The formalism to treat fermions in the Schrödinger picture with the help of functionals depending on Grassmann variables can be found, for example, in Refs.<sup>17,20</sup>.

Working in the temporal axial gauge for the electromagnetic gauge potential,  $A^0 = 0$ , we begin with the classical Hamiltonian:

$$H[\pi, \pi^*, \vec{\Pi}, \Phi, \Phi^*, \vec{A}] = \int d^3x \left\{ \pi \pi^* + \vec{D}\Phi \cdot (\vec{D}\Phi)^* + \frac{1}{2} (\vec{\Pi}^2 + \vec{B}^2) + V(\Phi \Phi^*) \right\} , \quad (32)$$

describing the charged (complex) scalar field  $\Phi$  in interaction with the gauge field  $\vec{A}$  and a quadratic selfinteraction term  $V$ ; the covariant derivative  $\vec{D}_\mu$  is as in Eq. (4). There is the magnetic field contribution  $\vec{B}^2 = \frac{1}{2} F_{ij} F^{ij}$ ,  $i, j = 1, 2, 3$ ,  $F_{ij} \equiv \partial_i A_j - \partial_j A_i$ , and the canonical momenta  $\pi, \pi^*, \vec{\Pi}$  are obtained from the corresponding Lagrangian. Since  $A^0$  is not dynamical, the Hamiltonian has to be supplemented by Gauss' Law:

$$\nabla \cdot \vec{\Pi} = -ie (\Phi \pi - \Phi^* \pi^*) . \quad (33)$$

We recall the relation with the electric field,  $\Pi^i = E_i \equiv F_{0i} \equiv \partial_0 A_i$ , in this gauge.

The theory is quantized in the coordinate representation (cf. Section 3.1) by:

$$\pi^{(*)} \longrightarrow \hat{\pi}^{(\dagger)} \equiv \frac{1}{i} \frac{\delta}{\delta \Phi^{(*)}} , \quad (34)$$

$$\Pi_j \longrightarrow \hat{\Pi}_j \equiv -\frac{1}{i} \frac{\delta}{\delta A^j} , \quad (35)$$

which, in particular, implies the commutation relation:  $[A^j(\vec{x}), \Pi_k(\vec{y})] = i\delta_k^j \delta^3(\vec{x} - \vec{y})$ . Furthermore, then, the quantized version of Gauss' Law is obtained:

$$\left\{ \partial^j \frac{\delta}{\delta A^j} - ie \left( \Phi \frac{\delta}{\delta \Phi} - \Phi^* \frac{\delta}{\delta \Phi^*} \right) \right\} \Psi[\Phi, \Phi^*, \vec{A}; t] = 0 . \quad (36)$$

It expresses the invariance of the wave functional under U(1) gauge transformations:

$$\vec{A}'(\vec{x}) = \vec{A}(\vec{x}) - \nabla \lambda(\vec{x}) , \quad \Phi'(\vec{x}) = \Phi(\vec{x}) \exp[-ie\lambda(\vec{x})] , \quad (37)$$

with  $\lambda$  an arbitrary real function. So far, this presents the starting point for a study of the Abelian Higgs model or scalar QED in the functional Schrödinger picture<sup>19</sup>.

The question must be raised now, whether the electromagnetic U(1) symmetry can *coexist* with the new  $\mathcal{U}(1)$  symmetry, which we introduced in Section 3.1. The answer is positive, as we shall demonstrate in the following.

As before, we replace the time derivative and the functional derivatives, which appear in the Hamiltonian or in Gauss' Law, by covariant ones:

$$\partial_t \longrightarrow \mathcal{D}_t \equiv \partial_t + i\mathcal{A}_t[\Phi, \Phi^*, \vec{A}; t] , \quad (38)$$

$$\frac{\delta}{\delta \Xi(\vec{x})} \longrightarrow \mathcal{D}_{\Xi(\vec{x})} \equiv \frac{\delta}{\delta \Xi(\vec{x})} + i\mathcal{A}_{\Xi}[\Phi, \Phi^*, \vec{A}; t, \vec{x}] , \quad (39)$$

where  $\Xi$  represents any one of the fields  $\Phi, \Phi^*, \vec{A}$ , on which the wave functional depends. For each one, we presently need a separate ‘gauge potential’  $\mathcal{A}_\Xi$ , which is minimally coupled. This implements the covariance under the  $\mathcal{U}(1)$  transformations:

$$\Psi'[\tilde{\Xi}; t] = \exp(-i\Lambda[\tilde{\Xi}; t])\Psi[\tilde{\Xi}; t] , \quad (40)$$

$$\mathcal{A}'_t[\tilde{\Xi}; t] = \mathcal{A}_t[\tilde{\Xi}; t] + \partial_t \Lambda[\tilde{\Xi}; t] , \quad (41)$$

$$\mathcal{A}'_\Xi[\tilde{\Xi}; t, \vec{x}] = \mathcal{A}_\Xi[\tilde{\Xi}; t, \vec{x}] + \frac{\delta}{\delta \Xi(\vec{x})} \Lambda[\tilde{\Xi}; t] , \quad (42)$$

for the present model; we collectively denote  $\Phi, \Phi^*, \vec{A}$  by  $\tilde{\Xi}$  from now on. As before, we define an invariant ‘electric field strength’:

$$\mathcal{F}_{t\Xi}[\tilde{\Xi}; t, \vec{x}] \equiv \partial_t \mathcal{A}_\Xi[\tilde{\Xi}; t, \vec{x}] - \frac{\delta}{\delta \Xi(\vec{x})} \mathcal{A}_t[\tilde{\Xi}; t] , \quad (43)$$

one for each field. We may also define an invariant bilocal ‘magnetic field strenght’:

$$\mathcal{F}_{\Xi\Xi'}[\tilde{\Xi}; t, \vec{x}, \vec{y}] \equiv \frac{\delta}{\delta \Xi(\vec{x})} \mathcal{A}_{\Xi'}[\tilde{\Xi}; t, \vec{y}] - \frac{\delta}{\delta \Xi'(\vec{y})} \mathcal{A}_\Xi[\tilde{\Xi}; t, \vec{x}] , \quad (44)$$

which does not exist in the single scalar field case of Section 3.1.

Then, an  $\mathcal{U}(1)$  and  $U(1)$  invariant action for Abelian Higgs model or scalar QED is:

$$\begin{aligned} \Gamma \equiv & \int dt D\Phi D\Phi^* D\vec{A} \left\{ \Psi^*[\tilde{\Xi}; t] \left( \mathcal{N}(\rho) i\overleftrightarrow{\mathcal{D}}_t - H\left[\frac{1}{i}\mathcal{D}_\Xi, \tilde{\Xi}\right] \right) \Psi[\tilde{\Xi}; t] \right. \\ & \left. - \frac{l^2}{2} \sum_{\Xi} \int d^3x (\mathcal{F}_{t\Xi})^2 - \frac{l'}{4} \sum_{\Xi, \Xi'} \int d^3x d^3y (\mathcal{F}_{\Xi\Xi'})^2 \right\} , \end{aligned} \quad (45)$$

with Hamiltonian of Eq. (32), quantized according to Eqs. (34)–(35), and with time and functional derivatives replaced by covariant ones, Eqs. (38)–(39). For dimensional reasons, the ‘magnetic field squared’ contribution comes with a *length* parameter  $l'$ , which at this point can be chosen differently from  $\sqrt{l^2}$ .

We do not discuss the equations of motion here, which simply generalize the ones of Section 3.1. However, turning to the ‘Gauss’ Law’ which is due to the ‘potential’  $\mathcal{A}_t[\tilde{\Xi}; t]$  acting as a Lagrange multiplier, we presently obtain:

$$\sum_{\Xi} \int d^3x \frac{\delta}{\delta \Xi(\vec{x})} \mathcal{F}_{t\Xi}[\tilde{\Xi}; t, \vec{x}] = \frac{1}{l^2} \rho \mathcal{N}(\rho) , \quad (46)$$

with  $\rho \equiv \Psi^*[\tilde{\Xi}; t] \Psi[\tilde{\Xi}; t]$ . Thus, besides Eq. (36), we have a second constraint.

The two constraints respectively express  $\mathcal{U}(1)$  and  $U(1)$  gauge invariance. One way to arrange the coexistence of these symmetries consists in decoupling the respective constraints. – Having replaced the functional derivatives in Eq. (36) by the covariant ones from Eq. (39), let the  $\mathcal{A}_\Xi$ -functionals which enter be *pure gauge* with respect to  $\mathcal{U}(1)$ :

$$\mathcal{A}_\Xi[\tilde{\Xi}; t, \vec{x}] = \frac{\delta}{\delta \Xi(\vec{x})} \alpha[\tilde{\Xi}; t] , \quad (47)$$

with  $\alpha$  an  $U(1)$  invariant functional. In this case, their contributions to the ordinary Gauss Law (36) cancel, which is thus kept intact, while being invariant under  $\mathcal{U}(1)$ . If we, furthermore, restrict  $\mathcal{A}_t[\tilde{\Xi}; t]$  to be  $U(1)$  invariant, then this is also true of the ‘Gauss Law’ (46), since the right-hand side is invariant already by Eq. (36). In this restricted situation, the ‘magnetic field’ (44) vanishes. Correspondingly, the ‘magnetic’ contribution to the action  $\Gamma$ , Eq. (45), and the dependence on the length parameter  $l'$  disappear. What is left is the minimally coupled  $\mathcal{U}(1)$  dynamics, incorporating only  $\mathcal{A}_t$ , which does not interfere with the internal  $U(1)$  symmetry of the Abelian Higgs model or scalar QED.

Nothing prevents the extension of this construction to more realistic models, incorporating non-Abelian gauge symmetries, for example.

## 4 Conclusions

Gauge transformations “of the third kind” which attribute a  $\mathcal{U}(1)$  ‘charge’ to the wave functional have been introduced here. This leads to an embedding of quantum field theory in a larger nonlinear structure, which differs from the earlier proposals of nonlinear generalizations of quantum mechanics or QFT<sup>3,4,5,6</sup>.

We tentatively interpret it as a classical one, since real and imaginary part of the wave functional  $\Psi$  are related to its differently charged components which, besides being governed by the interactions of the underlying field theory model, are coupled through a new connection functional  $\mathcal{A}$ . When effects of the latter are negligible, QFT is recovered.

A number of interesting problems clearly need further study, before this proposal can stand on its own:

- A theory of the observables and the measurement process needs to be worked out. It seems promising that the energy-momentum tensor following from our action, Eq. (14) or Eq. (45), is the one of a quantized scalar or Abelian Higgs model (scalar QED), respectively, *plus* contributions due to the coupling between  $\Psi$  and  $\mathcal{A}$ . When the latter is small, the usual observables might be useful, while the coupling might be important for the reduction or collapse of the wave functional.
- The symmetry properties of our extended theory need to be considered in detail, particularly in view of the introduction of the fundamental length  $l^a$ .
- A solution in the case of an underlying free field theory should be possible, based on the variational principle, for example. This will be helpful to better understand the new coupling and induced nonlocal correlations.
- Besides the indicated extension for coupled gauge and matter fields, reparametrization invariant models are an important target. As compared to a Wheeler-DeWitt type equation, giving rise to the “problem of time” and ensuing difficulties to understand evolution in a frozen picture, the presence of additional nonlinear terms in what replaces this equation might actually be useful.
- Is there more to the choice of the necessary nonlinearity  $\mathcal{N}(\Psi^*\Psi)$ , Eq. (22), and its relation to an entropy/area parameter<sup>22</sup> and information?

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<sup>a</sup>Under suitable assumptions about the background metric, the action  $\Gamma$  can be written in explicitly Lorentz invariant form, to be discussed elsewhere.

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